Filomat 29:6 (2015), 1307–1314
DOI 10.2298/FIL1506307S

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

On Some Generalized Integral Inequalities for Riemann-Liouville Fractional Integrals

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Abstract. In this paper, we give a generalized Montogomery identities for the Riemann-Liouville fractional integrals. We also use this Montogomery identities to establish some new Ostrowski type integral inequalities.

1. Introduction

The inequality of Ostrowski [20] gives us an estimate for the deviation of the values of a smooth function from its mean value. More precisely, if $f : [a, b] \to \mathbb{R}$ is a differentiable function with bounded derivative, then

$$\left| f(x) - \frac{1}{b-a}\int_a^b f(t)dt \right| \leq \left[\frac{1}{4} + \frac{(x-\frac{a+b}{2})^2}{(b-a)^2} \right](b-a)\left\| f' \right\|_\infty$$

for every $x \in [a, b]$. Moreover the constant $1/4$ is the best possible.

For some generalizations of this classic fact see the book [9, p.468-484] by Mitrinovic, Pecaric and Fink. A simple proof of this fact can be done by using the following identity [9]:

If $f : [a, b] \to \mathbb{R}$ is differentiable on $[a, b]$ with the first derivative f' integrable on $[a, b]$, then Montgomery identity holds:

$$f(x) = \frac{1}{b-a}\int_a^b f(t)dt + \int_a^b P_1(x,t) f'(t)dt, \tag{1}$$

where $P_1(x, t)$ is the Peano kernel defined by

$$P_1(x,t) := \begin{cases} \dfrac{t-a}{b-a}, & a \leq t < x \\[2ex] \dfrac{t-b}{b-a}, & x \leq t \leq b. \end{cases}$$

2010 *Mathematics Subject Classification*. Primary 26D15; Secondary 41A55, 26D10
Keywords. Riemann-Liouville fractional integral, Ostrowski inequality
Received: 03 December 2013; Accepted: 28 March 2014
Communicated by Hari M. Srivastava
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Recently, several generalizations of the Ostrowski integral inequality are considered by many authors; for instance covering the following concepts: functions of bounded variation, Lipschitzian, monotonic, absolutely continuous and n-times differentiable mappings with error estimates with some special means together with some numerical quadrature rules. For recent results and generalizations concerning Ostrowski's inequality, we refer the reader to the recent papers [3], [6], [10]-[12], [14]-[16].

In [1] and [17], the authors established some inequalities for differentiable mappings which are connected with Ostrowski type inequality by used the Riemann-Liouville fractional integrals, and they used the following lemma to prove their results:

Lemma 1.1. *Let $f : I \subset \mathbb{R} \to \mathbb{R}$ be differentiable function on I° with $a, b \in I$ $(a < b)$ and $f' \in L_1[a, b]$, then*

$$f(x) = \frac{\Gamma(\alpha)}{b-a}(b-x)^{1-\alpha} J_a^\alpha f(b) - J_a^{\alpha-1}(P_2(x,b)f(b)) + J_a^\alpha(P_2(x,b)f'(b)), \quad \alpha \geq 1, \tag{2}$$

where $P_2(x, t)$ is the fractional Peano kernel defined by

$$P_2(x,t) = \begin{cases} \dfrac{t-a}{b-a}(b-x)^{1-\alpha}\Gamma(\alpha), & a \leq t < x \\[2ex] \dfrac{t-b}{b-a}(b-x)^{1-\alpha}\Gamma(\alpha), & x \leq t \leq b. \end{cases}$$

In this article, we use the Riemann-Liouville fractional integrals to establish some new integral inequalities of Ostrowski's type.

2. Fractional Calculus

Firstly, we give some necessary definitions and mathematical preliminaries of fractional calculus theory which are used further in this paper. More details, one can consult [7], [8], [13].

Definition 2.1. *The Riemann-Liouville fractional integral operator of order $\alpha \geq 0$ with $a \geq 0$ is defined as*

$$\begin{aligned} J_a^\alpha f(x) &= \frac{1}{\Gamma(\alpha)} \int_a^x (x-t)^{\alpha-1} f(t)dt, \\ J_a^0 f(x) &= f(x). \end{aligned}$$

Recently, many authors have studied a number of inequalities by used the Riemann-Liouville fractional integrals, see ([1], [2], [4], [5], [17], [18], [19]) and the references cited therein.

3. Main Results

Throughout this work, we assume that Peano kernels defined by

$$K_1(x,t) = \begin{cases} \left[t-a-\frac{\lambda}{2}(x-a)\right] & , a \leq t < x \\[2ex] \left[t-b+\frac{\lambda}{2}(b-x)\right] & , x \leq t \leq b \end{cases}$$

$$K_2(x,t) = \begin{cases} \frac{1}{b-a}\left[t-a-\frac{\lambda}{2}(x-a)\right](b-x)^{1-\alpha}\Gamma(\alpha) & , a \leq t < x \\[2ex] \frac{1}{b-a}\left[t-b+\frac{\lambda}{2}(b-x)\right](b-x)^{1-\alpha}\Gamma(\alpha) & , x \leq t \leq b \end{cases}$$

$$h(x,t) = \begin{cases} \frac{1}{b-a}\left[t-a+\frac{\lambda}{2}(x-a)\right](b-x)^{1-\alpha}\Gamma(\alpha) & , a \leq t < x \\[2ex] \frac{1}{b-a}\left[b-t+\frac{\lambda}{2}(b-x)\right](b-x)^{1-\alpha}\Gamma(\alpha) & , x \leq t \leq b \end{cases}$$

In order to prove our main results, we need the following lemma:

Lemma 3.1. *Let $f : I \subset \mathbb{R} \to \mathbb{R}$ be a differentiable function on I° with $a, b \in I$ $(a < b)$, $\alpha \geq 1$, $0 \leq \lambda \leq 1$ and $f' \in L_1[a,b]$, then the generalization of Montgomery identity for fractional integrals holds:*

$$\begin{aligned}\left(1-\frac{\lambda}{2}\right) f(x) &= J_a^\alpha\left(K_2(x,b) f'(b)\right) + \frac{(b-x)^{1-\alpha}}{b-a}\Gamma(\alpha) J_a^\alpha f(b) \\ &\quad - J_a^{\alpha-1}\left(K_2(x,b) f(b)\right) - \frac{\lambda}{2}(b-a)^{\alpha-2}(x-a)(b-x)^{\alpha-1} f(a)\end{aligned} \tag{3}$$

Proof. By definition of $K_1(x,t)$ and integrating by parts, we can state:

$$\begin{aligned}
&\Gamma(\alpha) J_a^\alpha\left(K_1(x,b) f'(b)\right) \\
&= \int_a^b (b-t)^{\alpha-1} K_1(x,t) f'(t)\,dt \\
&= \int_a^x (b-t)^{\alpha-1}\left(t-a-\frac{\lambda}{2}(x-a)\right) f'(t)\,dt + \int_x^b (b-t)^{\alpha-1}\left(t-b+\frac{\lambda}{2}(b-x)\right) f'(t)\,dt \\
&= (b-t)^{\alpha-1}\left(t-a-\frac{\lambda}{2}(x-a)\right) f(t)\Big|_a^x + (b-t)^{\alpha-1}\left(t-b+\frac{\lambda}{2}(b-x)\right) f(t)\Big|_x^b \\
&- \int_a^x \left[(b-t)^{\alpha-1} - (\alpha-1)(b-t)^{\alpha-2}\left(t-a-\frac{\lambda}{2}(x-a)\right)\right] f(t)\,dt \\
&- \int_x^b \left[(b-t)^{\alpha-1} - (\alpha-1)(b-t)^{\alpha-2}\left(t-b+\frac{\lambda}{2}(b-x)\right)\right] f(t)\,dt \\
&= (b-x)^{\alpha-1}(x-a)\left(1-\frac{\lambda}{2}\right) f(x) + \frac{\lambda}{2}(b-a)^{\alpha-1}(x-a) f(a) + (b-x)^{\alpha}\left(1-\frac{\lambda}{2}\right) f(x) \\
&- \int_a^b (b-t)^{\alpha-1} f(t)\,dt + (\alpha-1)\int_a^b (b-t)^{\alpha-2} K_1(x,t) f(t)\,dt \\
&= \frac{\lambda}{2}(b-a)^{\alpha-1}(x-a) f(a) + \left(1-\frac{\lambda}{2}\right)(b-x)^{\alpha-1}(b-a) f(x) \\
&- \int_a^b (b-t)^{\alpha-1} f(t)\,dt + (\alpha-1)\int_a^b (b-t)^{\alpha-2} K_1(x,t) f(t)\,dt.
\end{aligned}$$

Finally, from definition of $K_2(x,t)$, we obtain the following identity

$$\begin{aligned}\left(1-\frac{\lambda}{2}\right) f(x) &= \frac{(b-x)^{1-\alpha}\Gamma(\alpha)}{b-a} J_a^\alpha\left(P_1(x,b) f'(b)\right) \\ &\quad + \frac{(b-x)^{1-\alpha}}{b-a}\frac{\Gamma(\alpha)}{\Gamma(\alpha)}\int_a^b (b-t)^{\alpha-1} f(t)\,dt\end{aligned}$$

$$-\frac{(b-x)^{\alpha-1}}{b-a}\frac{(\alpha-1)\Gamma(\alpha-1)}{\Gamma(\alpha-1)}\int_a^b (b-t)^{\alpha-2}K_1(x,t)f(t)\,dt$$

$$-\frac{\lambda}{2}(b-a)^{\alpha-1}\frac{(x-a)(b-x)^{1-\alpha}}{b-a}f(a)$$

$$= J_a^{\alpha}\left(K_2(x,b)f'(b)\right)+\frac{(b-x)^{1-\alpha}}{b-a}\Gamma(\alpha)J_a^{\alpha}f(b)$$

$$-J_a^{\alpha-1}(K_2(x,b)f(b))-\frac{\lambda}{2}(b-a)^{\alpha-2}(x-a)(b-x)^{\alpha-1}f(a)$$

which this completes the proof. □

Remark 3.2. *Letting $\alpha=1$ and $\lambda=0$, the formula (3) reduces to the classical Montgomery Identity given by (1).*

Remark 3.3. *If we choose $\lambda=0$, the formula (3) reduces to the fractional Montgomery Identity given by (2) proved in [1] and [17].*

Corollary 3.4. *Under assumption of Lemma 3.1 with $\lambda=1$, we have*

$$f(x) = 2J_a^{\alpha}\left(K_2(x,b)f'(b)\right)+\frac{2(b-x)^{1-\alpha}}{b-a}\Gamma(\alpha)J_a^{\alpha}f(b)$$

$$-2J_a^{\alpha-1}(K_2(x,b)f(b))-(b-a)^{\alpha-2}(x-a)(b-x)^{\alpha-1}f(a).$$

Theorem 3.5. *Let $f:[a,b]\to\mathbb{R}$ be differentiable on (a,b) such that $f'\in L_1[a,b]$, where $a<b$ and $0\le\lambda\le 1$. If $\left|f'(x)\right|\le M$ for every $x\in[a,b]$ and $\alpha\ge 1$, then the following Ostrowski fractional inequality holds:*

$$\left|\left(1-\frac{\lambda}{2}\right)f(x)-\frac{(b-x)^{1-\alpha}}{b-a}\Gamma(\alpha)J_a^{\alpha}f(b)+J_a^{\alpha-1}(K_2(x,b)f(b))+\frac{\lambda}{2}(b-a)^{\alpha-2}(x-a)(b-x)^{\alpha-1}f(a)\right| \tag{4}$$

$$\le\frac{M}{\Gamma(\alpha)}A(x)$$

where

$$A(x) = \frac{\Gamma(\alpha)(b-x)^{1-\alpha}}{(b-a)}\left\{(b-a)^{\alpha}\left[\frac{2(b-a)+\lambda(x-a)}{2\alpha}-\frac{b-a}{\alpha+1}\right]+(b-x)^{\alpha}\left[\frac{2(b-x)}{\alpha+1}-\frac{(b-a)+\lambda\left(x-\frac{a+b}{2}\right)}{\alpha}\right]\right\}.$$

Proof. From Lemma 3.1, we get

$$\left|\left(1-\frac{\lambda}{2}\right)f(x)-\frac{(b-x)^{1-\alpha}}{b-a}\Gamma(\alpha)J_a^{\alpha}f(b)+J_a^{\alpha-1}(K_2(x,b)f(b))\right.$$

$$\left.+\frac{\lambda}{2}(b-a)^{\alpha-2}(x-a)(b-x)^{\alpha-1}f(a)\right|\le\frac{1}{\Gamma(\alpha)}\left|\int_a^b(b-t)^{\alpha-1}K_2(x,t)f'(t)\,dt\right| \tag{5}$$

$$\le\frac{M}{\Gamma(\alpha)}\int_a^b(b-t)^{\alpha-1}|K_2(x,t)|\,dt$$

$$\le\frac{M}{\Gamma(\alpha)}\int_a^b(b-t)^{\alpha-1}h(x,t)\,dt.$$

Therefore, by simple computation of integral, we have

$$\frac{1}{\Gamma(\alpha)}\int_a^b (b-t)^{\alpha-1} h(x,t)dt \tag{6}$$

$$= \frac{(b-x)^{1-\alpha}}{(b-a)}\left\{\int_a^x (b-t)^{\alpha-1}\left(t-a+\frac{\lambda}{2}(x-a)\right)dt+\int_x^b (b-t)^{\alpha-1}(b-t+\frac{\lambda}{2}(b-x))dt\right\}$$

$$= \frac{(b-x)^{1-\alpha}}{(b-a)}\left\{(b-a)^{\alpha}\left[\frac{2(b-a)+\lambda(x-a)}{2\alpha}-\frac{b-a}{\alpha+1}\right]\right.$$

$$\left.+(b-x)^{\alpha}\left[\frac{2(b-x)}{\alpha+1}-\frac{(b-a)+\lambda\left(x-\frac{a+b}{2}\right)}{\alpha}\right]\right\}.$$

Using (6) in (5), we obtain (4). This proves inequality (4). □

Remark 3.6. *We note that in the special cases, if we take $\lambda = 0$ in Theorem 3.5, then it reduces Theorem 4.1 proved by Anastassiou et. al. [1]. So, our results are generalizations of the corresponding results of Anastassiou et. al. [1].*

Corollary 3.7. *Under the assumptions of Theorem 3.5 with $\lambda = 1$, we have*

$$\left|\frac{1}{2}f(x)-\frac{(b-x)^{1-\alpha}}{b-a}\Gamma(\alpha)J_a^{\alpha}f(b)+J_a^{\alpha-1}(K_2(x,b)f(b))\right. \tag{7}$$

$$\left.+\frac{1}{2}(b-a)^{\alpha-2}(x-a)(b-x)^{\alpha-1}f(a)\right|$$

$$\leq \frac{M}{\alpha(\alpha+1)}\left\{(b-x)^{1-\alpha}(b-a)^{\alpha-1}[2(b-a)+\alpha(\alpha+1)(x-a)]\right.$$

$$\left.+\frac{(b-x)}{(b-a)}[3\alpha(a+b-2x)-(b-a)]\right\}.$$

Remark 3.8. *Letting $\alpha = 1$, the inequality (7) reduces to*

$$\left|\frac{(b-a)f(x)+(x-a)f(a)}{2(b-a)}-\frac{1}{b-a}\int_a^b f(t)dt\right|$$

$$\leq M\left\{(b-2a+x)+\frac{(b-x)}{(b-a)}(2a+b-3x)\right\}.$$

Theorem 3.9. *Let $f:[a,b]\to\mathbb{R}$ be differentiable on (a,b) such that $f'\in L_1[a,b]$, where $a<b$, $0\leq\lambda\leq 1$ and $\alpha\geq 1$. If the mapping $|f'|^q$ is convex on $[a,b]$, $q\geq 1$, then the following fractional inequality holds:*

$$\left|\left(1-\frac{\lambda}{2}\right)f(x)-\frac{(b-x)^{1-\alpha}}{b-a}\Gamma(\alpha)J_a^{\alpha}f(b)+J_a^{\alpha-1}(K_2(x,b)f(b))\right.$$

$$\left.+\frac{\lambda}{2}(b-a)^{\alpha-2}(x-a)(b-x)^{\alpha-1}f(a)\right| \tag{8}$$

$$\leq \frac{1}{\Gamma(\alpha)}(A(x))^{1-\frac{1}{q}}\left(|f'(a)|^q B(x)+|f'(b)|^q C(x)\right)^{\frac{1}{q}}$$

where

$$\begin{aligned} B(x) &= \frac{\Gamma(\alpha)(b-x)^{1-\alpha}}{(b-a)^2}\left\{(b-a)^{\alpha+1}\left[\frac{2(b-a)+\lambda(x-a)}{2(\alpha+1)}-\frac{b-a}{\alpha+2}\right]\right. \\ &\left. + (b-x)^{\alpha+1}\left[\frac{2(b-x)}{\alpha+2}-\frac{(b-a)+\lambda\left(x-\frac{a+b}{2}\right)}{\alpha+1}\right]\right\} \end{aligned}$$

and

$$\begin{aligned} C(x) &= \frac{\Gamma(\alpha)(b-x)^{1-\alpha}}{(b-a)}\left\{(b-a)^{\alpha}\left[\frac{2(b-a)+\lambda(x-a)}{2\alpha(\alpha+1)}-\left(\frac{b-a}{\alpha+1}-\frac{1}{(\alpha+2)}\right)\right]\right. \\ &+2(b-x)^{\alpha+1}\left(\frac{1}{\alpha+1}-\frac{(b-x)}{(\alpha+2)(b-a)}\right) \\ &\left.-(b-x)^{\alpha}\left((b-a)+\lambda\left(x-\frac{a+b}{2}\right)\right)\left(\frac{(b-x)}{(b-a)(\alpha+1)}-\frac{1}{\alpha}\right)\right\}. \end{aligned}$$

Proof. From Lemma 3.1, we get

$$\begin{aligned} &\left|\left(1-\frac{\lambda}{2}\right)f(x)-\frac{(b-x)^{1-\alpha}}{b-a}\Gamma(\alpha)J_a^{\alpha}f(b)+J_a^{\alpha-1}(K_2(x,b)f(b))+\frac{\lambda}{2}(b-a)^{\alpha-2}(x-a)(b-x)^{\alpha-1}f(a)\right| \\ &\leq \frac{1}{\Gamma(\alpha)}\left|\int_a^b (b-t)^{\alpha-1}K_2(x,t)f'(t)\,dt\right| \\ &\leq \frac{1}{\Gamma(\alpha)}\int_a^b (b-t)^{\alpha-1}|K_2(x,t)|\left|f'(t)\right|dt \\ &\leq \frac{1}{\Gamma(\alpha)}\int_a^b (b-t)^{\alpha-1}h(x,t)\left|f'(t)\right|dt. \end{aligned} \tag{9}$$

From Hölder's inequality and using convexity of $\left|f'\right|^q$, we have

$$\begin{aligned}
&\int_a^b (b-t)^{\alpha-1} h(x,t)\left|f'(t)\right| dt \\
&\leq \frac{1}{\Gamma(\alpha)}\left(\int_a^b (b-t)^{\alpha-1} h(x,t)\,dt\right)^{\frac{1}{p}}\left(\int_a^b (b-t)^{\alpha-1} h(x,t)\left|f'(t)\right|^q dt\right)^{\frac{1}{q}} \\
&= \frac{1}{\Gamma(\alpha)}\left(\int_a^b (b-t)^{\alpha-1} h(x,t)\,dt\right)^{1-\frac{1}{q}}\left(\int_a^b (b-t)^{\alpha-1} h(x,t)\left|f'\left(\frac{b-t}{b-a}a+\frac{t-a}{b-a}b\right)\right|^q dt\right)^{\frac{1}{q}} \\
&\leq \frac{1}{\Gamma(\alpha)}\left(\int_a^b (b-t)^{\alpha-1} h(x,t)\,dt\right)^{1-\frac{1}{q}}\left(\int_a^b (b-t)^{\alpha-1} h(x,t)\left[\frac{b-t}{b-a}\left|f'(a)\right|^q+\frac{t-a}{b-a}\left|f'(b)\right|^q\right]dt\right)^{\frac{1}{q}} \\
&= \frac{1}{\Gamma(\alpha)}\left(\int_a^b (b-t)^{\alpha-1} h(x,t)\,dt\right)^{1-\frac{1}{q}}\left(\frac{\left|f'(a)\right|^q}{b-a}\int_a^b (b-t)^{\alpha} h(x,t)\,dt+\frac{\left|f'(b)\right|^q}{b-a}\int_a^b (b-t)^{\alpha-1}(t-a) h(x,t)\,dt\right)^{\frac{1}{q}}
\end{aligned} \tag{10}$$

Now, by simple computation we get

$$\frac{\left|f'(a)\right|^q}{b-a}\int_a^b (b-t)^{\alpha} h(x,t)\,dt = B(x) \tag{11}$$

and similarly

$$\begin{aligned}
&\frac{\left|f'(b)\right|^q}{b-a}\int_a^b (b-t)^{\alpha-1}(t-a) h(x,t)\,dt \\
&= \left|f'(b)\right|^q\int_a^b (b-t)^{\alpha-1} h(x,t)\,dt - \frac{\left|f'(b)\right|^q}{b-a}\int_a^b (b-t)^{\alpha} h(x,t)\,dt = C(x)
\end{aligned} \tag{12}$$

Using (11) and (12) in (10), we obtain (8). This proves inequality (8). □

Corollary 3.10. *Under the assumptions of Theorem 3.9 with $\lambda = 1$, we have*

$$\begin{aligned}
&\left|\frac{1}{2}f(x) - \frac{(b-x)^{1-\alpha}}{b-a}\Gamma(\alpha) J_a^{\alpha} f(b) + J_a^{\alpha-1}\left(K_2(x,b) f(b)\right) + \frac{1}{2}(b-a)^{\alpha-2}(x-a)(b-x)^{\alpha-1} f(a)\right| \\
&\leq \frac{1}{\Gamma(\alpha)}\left(A_1(x)\right)^{1-\frac{1}{q}}\left(\left|f'(a)\right|^q B_1(x) + \left|f'(b)\right|^q C_1(x)\right)^{\frac{1}{q}}
\end{aligned} \tag{13}$$

where

$$A_1(x) = \frac{\Gamma(\alpha)(b-x)^{1-\alpha}}{(b-a)}\left\{(b-a)^{\alpha}\frac{2(b-a)+(\alpha+1)(x-a)}{2\alpha(\alpha+1)} + (b-x)^{\alpha}\frac{(\alpha-1)(b-a)+(a-x)-(x-a-b)}{\alpha(\alpha+1)}\right\}.$$

$$B_1(x) = \frac{\Gamma(\alpha)(b-x)^{1-\alpha}}{(b-a)^2}\left\{(b-a)^{\alpha+1}\frac{(b-a)+(\alpha+2)(x-a)}{2(\alpha+1)(\alpha+2)}\right.$$
$$\left. + (b-x)^{\alpha+1}\frac{\alpha(b-a)+2(\alpha+1)(a-x)-(x-a-b)}{(\alpha+1)(\alpha+2)}\right\}$$

and

$$C_1(x) = \frac{\Gamma(\alpha)(b-x)^{1-\alpha}}{(b-a)}\left\{(b-a)^{\alpha}\left[\frac{2(1-\alpha)(b-a)+(x-a)}{2\alpha(\alpha+1)}+\frac{1}{(\alpha+2)}\right]\right.$$
$$+2(b-x)^{\alpha+1}\frac{(b-x)+(\alpha+2)(x-a)}{(\alpha+2)(b-a)}$$
$$\left.-(b-x)^{\alpha}\left((b-a)+\left(x-\frac{a+b}{2}\right)\right)\left(\frac{(b-x)}{(b-a)(\alpha+1)}-\frac{1}{\alpha}\right)\right\}.$$

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